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Static Analysis of Mooring Lines for Station keeping of Offshore Structures

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ABSTRACT
Static analysis of mooring lines for the purpose of stationkeeping of offshore structures is of concern in this dissertation. Considering the fact that oil and gas exploration activities is drifting towards more and more deepwaters, the use of floating offshore structures is as well increasing. These floating structures must be kept under watch within a certain circle of operation (stationkeeping). Environmental forces such as wave and wind force acts on the structures and tend to move it away from the watch circle. Mooring lines are required to keep the structures within the desired excursion. The mooring lines must be designed to meet this requirement in terms of the tension in the cable (strength of the cable) as well as the length of the cable. The method considered for the static analysis of mooring lines in this work is the catenary equations. The equations were applied to both the elastic and inelastic cables. Detailed algorithm is developed for the implementation of the catenary equations for elastic and inelastic mooring lines. The developed algorithms are implemented using MATLAB. The developed MATLAB program is used to analyze two different test cases of mooring lines. Two other softwares (STAMOORSYS and LINANL) are also used to analyze the test cases. The maximum percentage difference between the results obtained from the developed MATLAB program and STAMOORSYS as well as LINANL are departure 0.05% and 0.09%, fairlead tension 0.0056% and 0.115%, suspended length 0.0056% and 0.114%, length on sea floor 0.007% and 0.7% respectively. As can be seen, the results obtained from the MATLAB program developed in this work showed insignificant deviation when compared with those of STAMOORSYS and LINANL. The MATLAB program is also used to analyze the cable line using the elastic and inelastic catenary model. The two models give similar results because of the high modulus of rigidity of the lines. It is recommended that dynamic models be developed to account for the dynamic response of the line since the static model ignores the effect of line motion.

Keywords: Mooring lines, Stationkeeping, Offshore Structures

I. INTRODUCTION

Over the years, floating offshore structures have extended to even deeper waters (2000 m+), and there has been a growing need for these structures to be stationed in these relatively deep waters with the accompanying hostile environmental conditions. This extension of floating offshore structures into deep waters is largely as a result of the identification of offshore oil as a potential source that can be harnessed to address the rising energy demand in the world. A study conducted by Chuku et al, (2018) based on the supposition that an offshore floating structure is designed and equipped with marine radar, AIS interface. The system is to be powered through a battery array from solar panels and transmit its signal through a satellite antenna to a control station otherwise referred as the base station. This floating structure is hinged on a mooring system which is very essentially anchored through accurate station keeping. They ensure that these floaters do not drift beyond an allowable range for safe operations. And this is to be achieved in spite of prevailing environmental conditions. The mooring line is one of the important components of the mooring system. Its behavior resembles that of a nonlinear spring. The tension-displacement characteristics of the mooring line depend on its length, weight, elastic properties, anchor holding capacities, and water depth. Based on this, several analysis methods have been made available for the analysis of mooring lines operating in shallow to deep water. As stated by [6], there are primarily three types of models for mooring line analysis: 1) catenary model, 2) lumped mass model and 3) finite element model. The catenary model (also known static method) is used to determine the static configuration or stiffness of the mooring line.

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It has many assumed conditions applied and thus many restrictions in use because environmental loads are neglected.

Exceeding this maximum allowable tension-displacement characteristic could result in the deformation of the mooring line and lead to the structures' excursion limits being exceeded. [1] stated that the drifting of an FPSO beyond its allowable limits leads to riser failure, disrupts the drilling operation and puts the safety of the FPSO in jeopardy. To ensure that the maximum allowable tension-displacement characteristics of a mooring line used in deep water is obtained and not exceeded, it is very necessary to analyze the behavior of such a mooring line taking into account the vessel load as a result of its motion and environmental loads.

This research work studies the approaches used in analyzing a mooring line, develops a suitable analytical mathematical model based on the identified approaches and utilizes the model for the static analysis of single component mooring line in deep water. It also develops an algorithm and a computer program using an appropriate programming language for carrying out mooring system analysis based on the developed model.The aim of this research work is to analyze the static response of a single component mooring system as a result of vessel's motion and environmental loads by obtaining its tension-displacement characteristics.This current research considers key objectives which includes to develop a proper analytical mathematical model for the representation of the entire mooring system, to implement the model into a computer application using MATLAB, STAMOORSYS and LINANL; and to validate the accuracy and suitability of the developed model and computer program using bench mark models.This analysis will provide information such as the configuration of the mooring system required for the given application, the tension-displacement characteristic of the mooring system and the suitability of the mooring system for that particular application.

Sensitivity analysis and adaptive learning approach could also deployed in this analysis, but the normalized percentage of each of the structures could pose difficulty and failure in obtaining results. This method is captured vividly in [2] where it asserts that the maritime industry stands to benefit from the approach's capability to monitor and manage critical energy systems during marine operations. This provides a guide/early warning against total failure of the critical ship systems.

II. MATERIALS AND METHOD

2.1 Catenary Model for Static Analysis of Mooring Lines

Mooring system requires suitable mathematical models and numerical techniques to successfully design the lines for station-keeping and integrity. The method considered for such design analysis is the catenary method. The method of static analysis which adopts the catenary mooring model applies the total steady environmental force due to wind and current to the load-excursion curve of the mooring system in order to find the static offset of the vessel.When a vessel is moored, the mooring line is anchored or pile to the seabed. The mooring line is slacked such that, some part of the line lies on the seabed. As a result of the hydrodynamic forces that act on the vessel, it drifts in the horizontal direction thereby causing the mooring line to be lifted from the seabed and a change in its configuration. The resultant lift and change in the angle of the mooring lines leads to increase in the tension of the mooring cable.The described behaviour of the mooring line can be modeled using the catenary equations to derive the line tensions and shape for any single line of a mooring pattern. The catenary equations are developed with the aid of the mooring line shown in Figure 1. In the development of the catenary equations several assumptions are made. The seabed is assumed to be horizontal. The cable is considered to be in the vertical plane and coincides with the x-z plane. Also, the bending stiffness of the cable is as well neglected. The latter is considered to be good approximation for chainsand also acceptable for large radius of curvature wires. The formulation of the catenary equations considered here also ignores the effect of the mooring line cable dynamics.

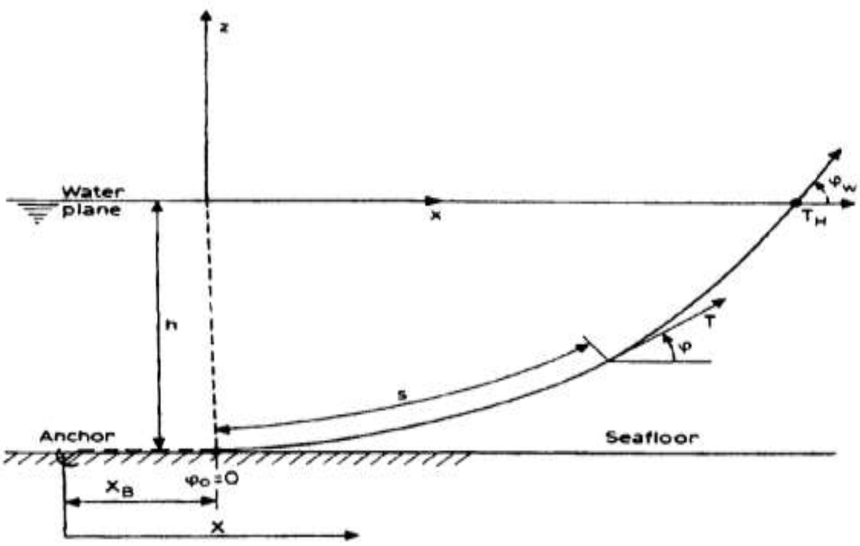

Figure 1:Catenary Mooring Line

In order to properly develop the catenary equations, we consider an element of the mooring line cable as depicted in Figure 2. As can be seen from the figure, w is defined as the weight per unit length of the cable when submerged in water. As also shown in the figure, the tangential and normal components of the mean hydrodynamic forces per unit length of the cable are denoted F and D respectively. The tension in the mooring line cable is denoted T, the cross sectional area of the line is denoted A and the modulus of elasticity of the mooring cable material is denoted E. The angle between the line and the horizontal plane is denoted φ.

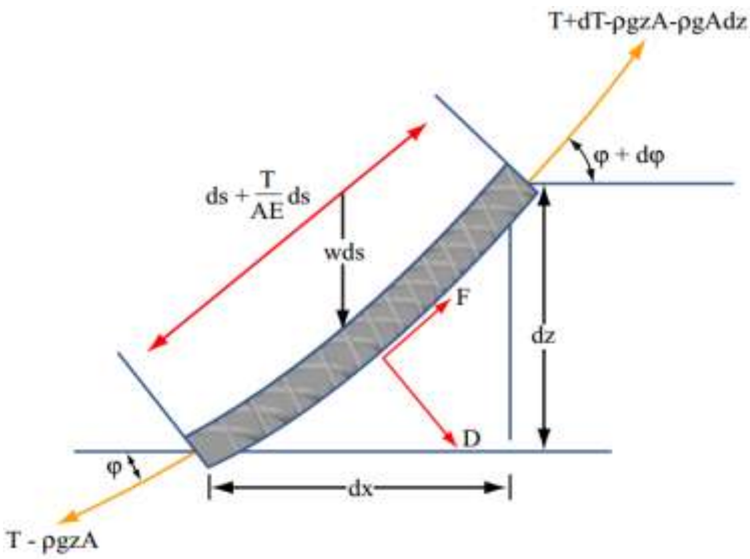

Figure 2: Element of Catenary Mooring Cable

Since w is considered the weight of the cable when submerged in water, to get the actual weight of the cable then, the weight of added water particle must be subtracted from the submerged weight of the cable in water. This result to the correction force of $-\rho gAz$ attached to the tensions at the ends of the element in Figure 2.

Considering static equilibrium of the forces acting on the element in Figure 2 and resolving all forces in the tangential direction:

$$(T + dT - \rho gzA - \rho gAdz) - (T - \rho gzA) = (wds \times \sin\varphi) - F\left(ds + \frac{T}{AE} ds\right) \quad (1)$$

$$dT - \rho gAdz = \left[w\sin\varphi - F\left(1 + \frac{T}{AE}\right)\right] ds \quad (2)$$

Similarly, resolving the forces in the normal direction, we can write the equilibrium equation as follows:

$$\mathrm{T d\varphi - \rho g A z d\varphi = (w ds \times \cos\varphi) + D\left(ds + \frac{T}{AE} ds\right)} \quad (3)$$

$$\mathrm{T d\varphi - \rho g A z d\varphi = \left[w \cos\varphi + D\left(1 + \frac{T}{AE}\right)\right] ds} \quad (4)$$

The catenary equations developed for static analysis of mooring lines presented as equations (2) and (4) are nonlinear in nature. As such, no explicit analytical solution can be obtained to solve these equations. These equations can only be solved numerically applying iterative methods.

2.2 Modeling of Inelastic Catenary Mooring Lines

The modeling of the catenary equations for inelastic lines follows a similar pattern in the previous section. The difference is that, the elastic component $\frac{T}{AE}ds$ is neglected. Also, to simplify the model, the drag and inertia component of the environmental load is also neglected. The diagram in Figure 3 shows the element used for the modeling of inelastic catenary mooring equations. Similarly to the foregoing section, the equilibrium equation of the mooring line element in Figure 3 is considered and the resultant forces acting on the element in the tangential direction is given as:

$$\mathrm{(T + dT - \rho g z A - \rho g A dz) - (T - \rho g z A) = (w ds \times \sin\varphi)} \quad (5)$$

$$\mathrm{dT - \rho g A dz = w \sin\varphi ds} \quad (6)$$

Similarly, resolving the forces in the normal direction, we can write the equilibrium equation as follows:

$$\mathrm{T d\varphi - \rho g A z d\varphi = (w ds \times \cos\varphi)} \quad (7)$$

$$\mathrm{T d\varphi - \rho g A z d\varphi = w \cos\varphi ds} \quad (8)$$

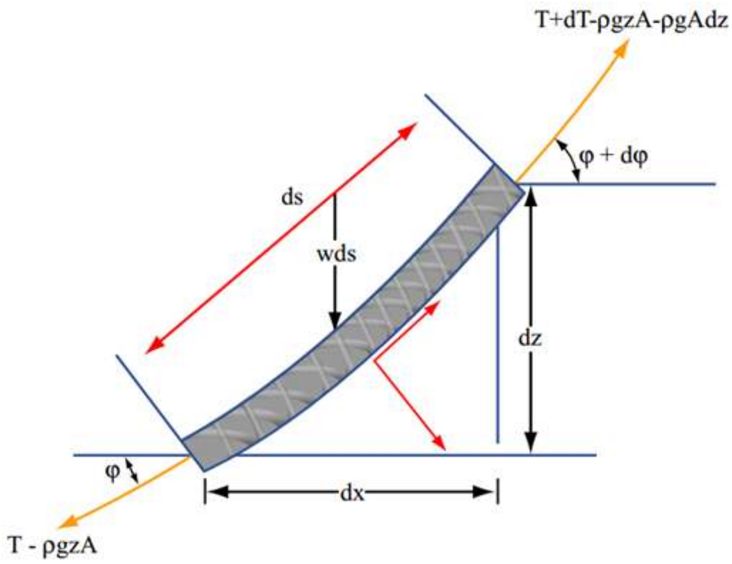

Figure 3: Element of Inelastic Catenary Mooring Cable

To further simplify the equations (6) and (8), we introduce the effective tension term($T' = T - \rho g z A$. Equations (6) and (8) can be further simplified as follows:

From equation (6)

$$dT' = w\sin\varphi ds \quad (9)$$

Similarly, from equation (3.8)

$$T' d\varphi = w\cos\varphi ds \quad (10)$$

Divide equation (3.9) by (3.10)

$$\frac{dT'}{T' d\varphi} = \frac{w\sin\varphi ds}{w\cos\varphi ds}$$

$$\frac{dT'}{T'} = \tan\varphi d\varphi \quad (11)$$

By integrating equation (3.11)

$$\int_{T'_0}^{T'} \frac{dT'}{T'} = \int_{\varphi_0}^{\varphi} \tan\varphi d\varphi$$

$$In\left(\frac{T'}{T'_0}\right) = In\left(\frac{\cos\varphi_0}{\cos\varphi}\right)$$

$$\frac{T'}{T'_0} = \frac{\cos\varphi_0}{\cos\varphi}$$

$$T' = T'_0 \frac{\cos\varphi_0}{\cos\varphi} \quad (12)$$

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From Figure 3 it can be obtained that $dz = ds \times sin\varphi$ and$dx = cos\varphi ds$. From equation (10);

$$T^{'}d\varphi = wcos\varphi ds$$

$$ds = \frac{T^{'}d\varphi}{wcos\varphi} = T^{'}_{0}\frac{cos\varphi_0}{cos\varphi} \times \frac{d\varphi}{wcos\varphi}$$

$$ds = T^{'}_{0}\frac{cos\,\varphi_0 d\varphi}{w cos^2\varphi} \qquad (13)$$

Integrate equation (3.13) to obtain an expression for s:

$$\int_{s_0}^{s} ds = \frac{T^{'}_{0}cos\,\varphi_0}{w}\int_{\varphi_0}^{\varphi}\frac{1}{cos^2\varphi}d\varphi$$

$$s - s_0 = \frac{T^{'}_{0}}{w}cos\varphi_0(tan\varphi - tan\varphi_0) \qquad (14)$$

Equation (13) can also be used to obtain an expression for z by equating$ds = \frac{dz}{sin\varphi}$, therefore;

$$ds = \frac{dz}{sin\varphi} = T^{'}_{0}\frac{cos\varphi_0 d\varphi}{wcos^2\varphi}$$

$$dz = T^{'}_{0}\frac{cos\,\varphi_0 sin\varphi}{w cos^2\varphi}d\varphi \qquad (15)$$

By integrating the equation (15)

$$\int_{z_0}^{z} dz = \frac{T^{'}_{0}cos\varphi_0}{w}\int_{\varphi_0}^{\varphi}\frac{sin\varphi}{cos^2\varphi}d\varphi$$

$$z - z_0 = \frac{T^{'}_{0}cos\varphi_0}{w}\left(\frac{1}{cos\varphi} - \frac{1}{cos\varphi_0}\right)$$

$$z - z_0 = \frac{T^{'}_{0}}{w}\left(\frac{cos\,\varphi_0 - cos\varphi}{cos\varphi}\right) \qquad (16)$$

From the relation$dx = cos\varphi ds$, we can derive a relation for x using equation (13).

$$dx = cos\varphi ds = T^{'}_{0}\frac{cos\varphi_0 d\varphi}{wcos^2\varphi}cos\varphi$$

$$dx = T^{'}_{0}\frac{cos\,\varphi_0}{wcos\varphi}d\varphi \qquad (17)$$

By integrating equation (17)

$$\int_{x_0}^{x} dx = \frac{T^{'}_{0}cos\varphi_0}{w}\int_{\varphi_0}^{\varphi}\frac{1}{cos\varphi}d\varphi$$

$$x - x_0 = \frac{T^{'}_{0}cos\,\varphi_0}{w}\left[In\left(\frac{1}{cos\varphi} + tan\varphi\right) - In1cos\varphi0 + tan\varphi0 \right. \qquad (18)$$

At this point, we apply the necessary boundary conditions on the developed equations. At the bottom of the mooring line (point 0), the point of contact between the line and the seabed; $\varphi_0 = 0$; $x_0 = 0$; $z_0 = -h$; therefore equation (3.12) give us

$$T^{'}_{0} = T^{'}cos\varphi$$

The horizontal component of the tension (T_H) at the waterplane may also be written as

$$T_H = Tcos\varphi_w$$

Therefore, it may be concluded by virtue of comparison that$T_H = T^{'}_{0}$. Equation (18) can further be simplified with the aid of the above stated boundary conditions and the horizontal tension as:

$$x - 0 = \frac{T_H cos0}{w}\left[In\left(\frac{1}{cos\varphi} + tan\varphi\right) - In\left(\frac{1}{cos0} + tan0\right)\right]$$

$$x = \frac{T_H}{w}In\left(\frac{1}{cos\varphi} + tan\varphi\right)$$

$$\frac{xw}{T_H} = In\left(\frac{1 + sin\varphi}{cos\varphi}\right)$$

$$sinh\left(\frac{xw}{T_H}\right) = \frac{1}{2}\left(\frac{1 + sin\varphi}{cos\varphi} - \frac{cos\varphi}{1 + sin\varphi}\right) = tan\varphi$$

$$cosh\left(\frac{xw}{T_H}\right) = \frac{1}{2}\left(\frac{1 + sin\varphi}{cos\varphi} + \frac{cos\varphi}{1 + sin\varphi}\right) = \frac{1}{cos\varphi}$$

Therefore, equation (3.14) can be further simplified to eliminate φ as follows:

$$s - 0 = \frac{T_H cos0}{w}(tan\varphi - tan0)$$

$$s = \frac{T_H}{w}tan\varphi$$

$$s = \frac{T_H}{w}sinh\left(\frac{xw}{T_H}\right) \qquad (19)$$

Similarly, equation (3.16) can be further simplified to eliminate φ as follows

$$z + h = \frac{T_H}{w}\left(\frac{cos\,0 - cos\varphi}{cos\varphi}\right) = \frac{T_H}{w}\left(\frac{1 - cos\varphi}{cos\varphi}\right) = \frac{T_H}{w}\left(\frac{1}{cos\varphi} - 1\right)$$

$$z + h = \frac{T_H}{w}\left[cosh\left(\frac{xw}{T_H}\right) - 1\right] \qquad (20)$$

The next item to be determined is the tension in the cable. This can be done using the relation $T^{'} = T - \rho gzA$and $T^{'}_{0} = T^{'}cos\varphi$; from these relations, the tension in the cable is determined as

$$T^{'} = T - \rho gzA = \frac{T_H}{cos\varphi}$$

$$T = T_H + wh + (w + \rho gh)z \qquad (21)$$

The vertical component of the tension in the cable line can be determined using the relation:

$$T_z = ws \qquad (22)$$

2.3 Development of Generalized Static Model for Elastic Catenary Mooring Lines

The inelastic catenary model neglects the deformation of the cable. It assumes that the cable is rigid. The procedure for modeling the elastic mooring cable is similar to that of the inelastic cable except the consideration of modulus of rigidity. Since the elastic cable is considered not to be rigid, [4] developed an approximate relationship for the stretched length and unstretched length of the mooring cable which is adopted in this work. The following relation was developed:

$$dp = ds\left(1 + \frac{T}{EA}\right) \quad (23)$$

$$\frac{dx}{ds} = \frac{dx}{dp} \times \frac{dp}{ds} = cos\varphi \times \left(1 + \frac{T}{EA}\right) \approx cos\varphi \times \left(1 + \frac{T'}{EA}\right) = cos\varphi + \frac{T_0'}{EA} \quad (24)$$

Similarly, we can also obtain a relationship for z as follows;

$$\frac{dz}{ds} = \frac{dz}{dp} \times \frac{dp}{ds} = sin\varphi \times \left(1 + \frac{T}{EA}\right) \approx sin\varphi \times \left(1 + \frac{T'}{EA}\right) = sin\varphi + \frac{ws}{EA} \quad (25)$$

The $\frac{ws}{EA}$ term in equation (25) is derived from the expressions $s = \frac{T_H}{w} tan\varphi$ and $T_H = Tcos\varphi$

Equations (24) and (25) are integrated in a similar manner as in the foregoing section and the necessary boundary conditions applied. From equation (25)

$$h = \frac{T_H}{W}\left(\frac{1}{cos\,\varphi_w} - 1\right) + \frac{1}{2}\frac{w}{EA} l_s^2 \quad (26)$$

where

l_s = length of the unstretched cable from where the cable touches the ground first.

$cos\varphi_w = \frac{T_H}{T}$ and $T = \left(T_H^2 + T_z^2\right)^{0.5}$

Therefore, the horizontal component of the tension can be obtained from equation (3.26) as

$$T_H = \frac{T_z^2 - \left(wh - \frac{1w^2}{2EA} l_s^2\right)^2}{2\left(wh - \frac{1w^2}{2EA} l_s^2\right)} \quad (27)$$

From the integration of equation (24)

$$x = \frac{T_H}{w} In\left[\frac{\left(T_H^2 + T_z^2\right)^{0.5} + T_z}{T_H}\right] + \frac{T_H}{EA} l_s \quad (28)$$

2.4 Algorithm for the Implementation Catenary Equations on Mooring Lines

2.4.1 Inelastic Catenary Mooring Lines Algorithm

For simplicity of the evaluation of the equations developed so far, the fairlead is assumed to be on the water level (i.e. z = 0) as shown in Figure 4. The steps highlighted below is the algorithm for analyzing inelastic mooring lines using the catenary equations.

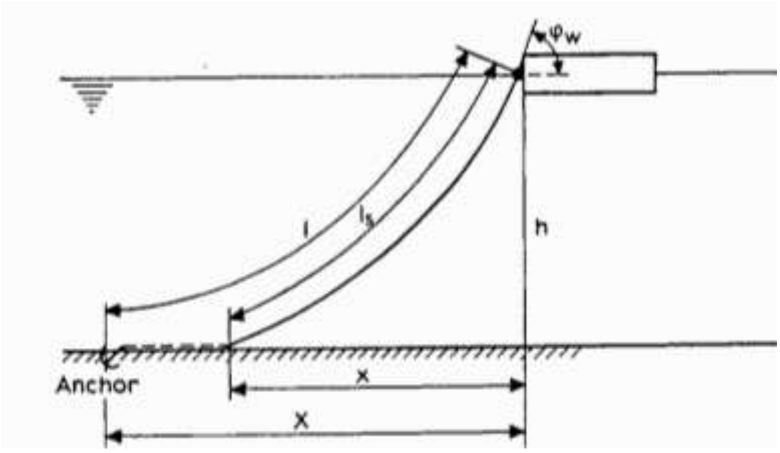

Figure 4: Moored Vessel on Seawater Level

Step 1: Specify the input parameters (horizontal tension (T_H), weight per unit length of the line (w), depth of the fairlead from the seabed (h) and the length of the line (l)).

Step 2: Compute the maximum tension of the cable using $T_{max} = T_H + wh$; hence, compute the minimum length of the line using $l_{min} = h\left(\frac{2a}{h} + 1\right)^{0.5}$; where $a = \frac{T_H}{w}$

Step 3: compute the horizontal distance from the touch down point to the fairlead using $x = acosh^{-1}\left(\frac{a}{h} + 1\right)$ from equation (3.20) where z = 0 as shown in Figure 4.

Step 4; compute the suspended length using $s = \frac{T_H}{w} sinh\left(\frac{xw}{T_H}\right)$

Step 5: compute the departure using the relation $X = l - s + x$

The flow chart for the developed pseudo code algorithm is depicted in Figure 5. A MATLAB program is written to implement the developed algorithm.

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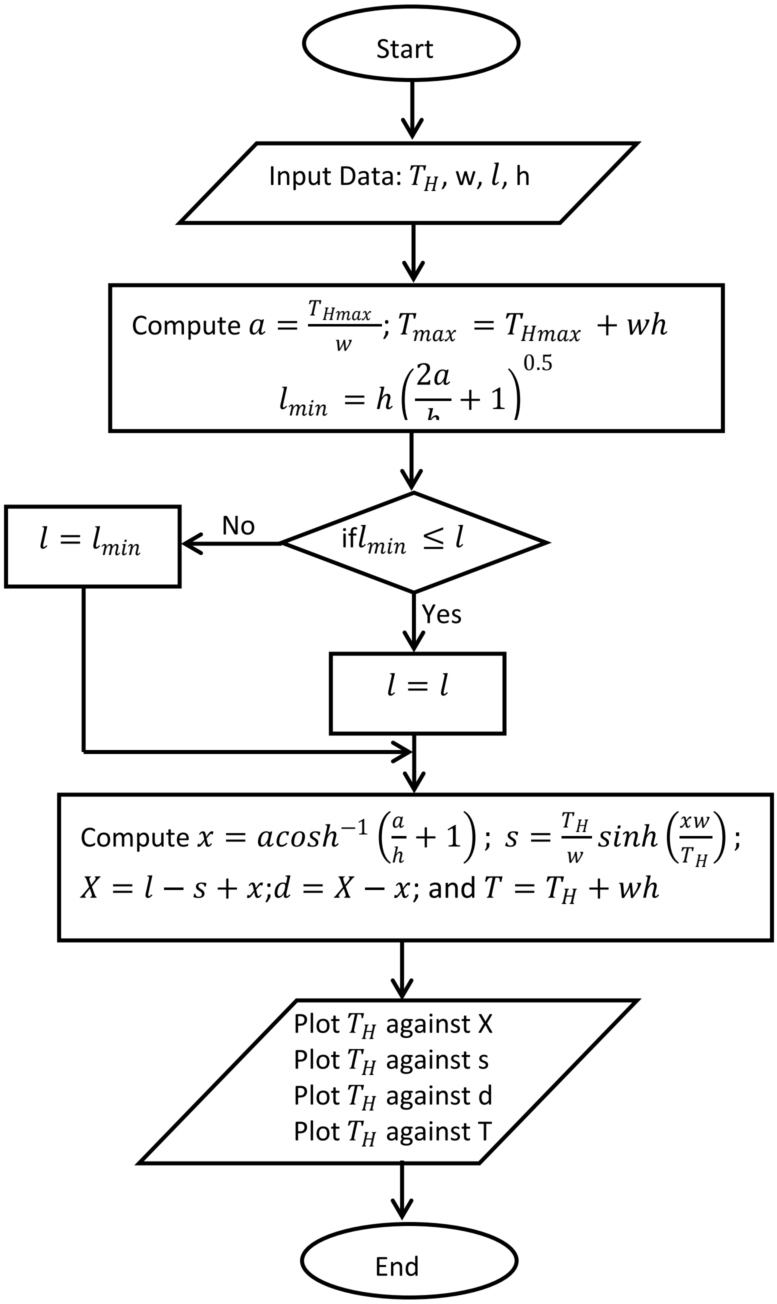

Figure 5: Flow Chart for Inelastic Catenary Mooring Lines

2.4.2 Elastic Catenary Mooring Lines Algorithm

Similarly, the elastic cable algorithm is developed following Figure 4. The equations developed for elastic cable is nonlinear in nature and required numerical approach in solving them. The approach adopted in this work is to assume the cable to be inelastic and obtain the inelastic solution. The detailed procedure is explained below:

Step 1: Compute the unstretched suspended length of the line using $l'_s = h\left(1 + \frac{2a}{h}\right)^{0.5}$ and the vertical tension of the unstretched line using $T'_z = wl'_s$

Step 2: Compute $T'_H = \frac{{T'_z}^2 - \left(wh - \frac{1w^2}{2EA}{l'_s}^2\right)^2}{2\left(wh - \frac{1w^2}{2EA}{l'_s}^2\right)}$

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Step 3: If $T'_H \neq T_H$; increase the value of T'_z and compute the value of $l'_s = \frac{T'_z}{w}$; hence, repeat step 2 until $T'_H = T_H$. Then, $l_s = l'_s$ and $T_z = T'_z$ for the stretched cable line.

Step 4: Compute $x = \frac{T_H}{w} In\left[\frac{(T_H{}^2+T_z{}^2)^{0.5}+T_z}{T_H}\right] + \frac{T_H}{EA} l_s$

Step 5: Compute the departure using the relation $X = l - s + x$

The flow chart for the developed pseudo code algorithm for elastic mooring lines is presented in Figure 6. A MATLAB program is also written to implement the developed algorithm

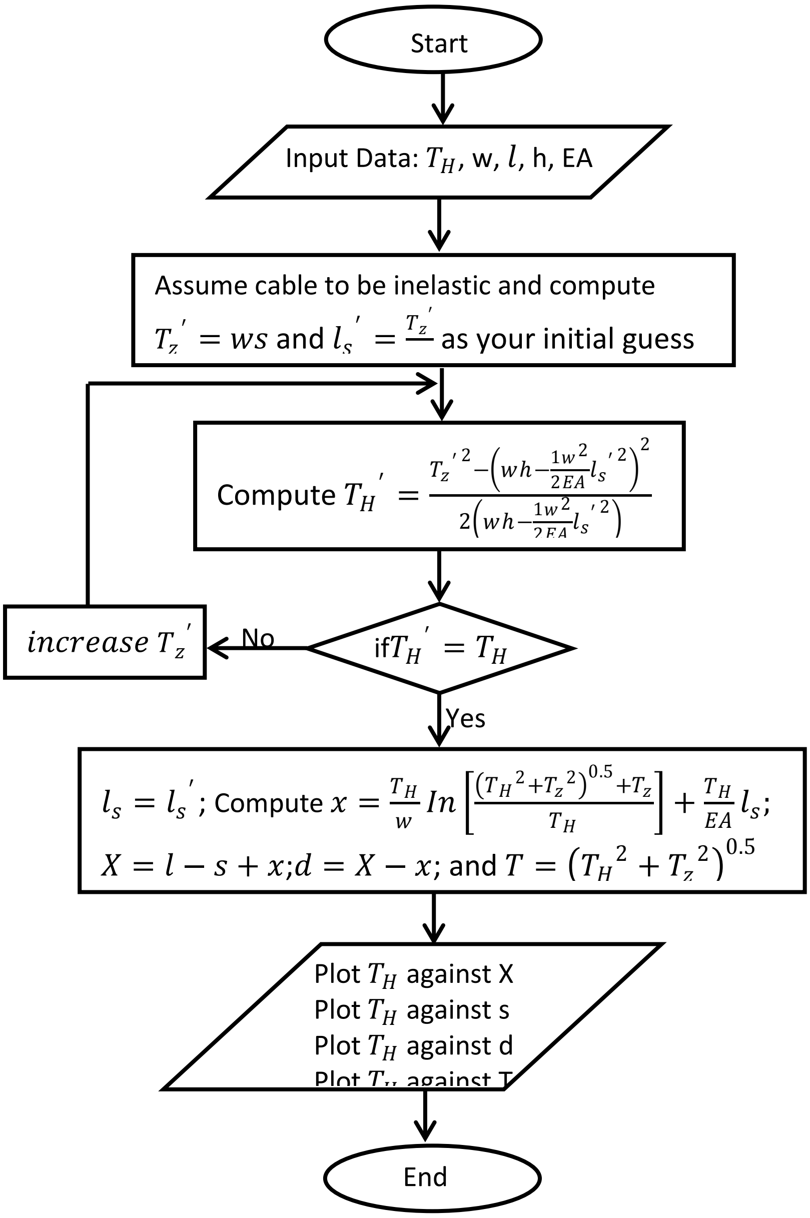

Figure 6: Flow Chart for Elastic Catenary Mooring Lines

2.5 Determination of the Mooring Stiffness Coefficient

The mooring stiffness coefficient also known as Hydrostatic restoring coefficient can be determined as the first derivative of the horizontal tension with respect to the departure$\left(\frac{dT_h}{dX}\right)$.

From the expressions $a = \frac{T_H}{w}; T_H = aw$ and $X = l - s + x$;

$$X = l - h\left(1+\frac{2a}{h}\right)^{0.5} + a\cosh^{-1}\left(\frac{h}{a}+1\right) \quad (29)$$

$$C_{11} = \frac{dT_h}{dX} = \frac{dT_h}{da}\times\frac{da}{dX} = w\left(\frac{dX}{da}\right)^{-1} \quad (30)$$

From equation (29);

$$\frac{dX}{da} = -\frac{h}{2}\left(1+\frac{2a}{h}\right)^{-0.5}\frac{h}{2} + \cosh^{-1}\left(\frac{h}{a}+1\right) + a\left\{\frac{-h}{a^2\left[\left(\frac{h}{a}+1\right)^2-1\right]^{0.5}}\right\}$$

$$= \cosh^{-1}\left(\frac{h}{a}+1\right) - \left(1+\frac{2a}{h}\right)^{-0.5} + \frac{h}{a}\left[\left(\frac{h}{a}+1\right)^2-1\right]^{0.5}$$

$$= \cosh^{-1}\left(\frac{h}{a}+1\right) - 2\left(1+\frac{2a}{h}\right)^{-0.5}$$

Therefore;

$$C_{11} = w\left(\cosh^{-1}\left(\frac{h}{a}+1\right) - 2\left(1+\frac{2a}{h}\right)^{-0.5}\right)^{-1} \quad (31)$$

[5]

III. RESULTS AND ANALYSIS

3.1 Test Case Data for Single Line Mooring (Validation of Developed Program)

A computer program is developed in this work for the implementation of the elastic and inelastic catenary models. The program is designed for the static analysis of single line mooring system. In order to validate the designed program, two different test cases were considered and analyzed.The two test cases were analyzed using the developed MATLAB program, STAMOORSYS and LINANL. LINANL is a software program designed for the static analysis of multiple-component mooring lines. STAMOORSYS is another software program developed by Texas A&M University for the static analysis of single and spread mooring system. The mooring lines considered in this analysis are divided into three segments as shown in Figure 7. The data of the various test cases are also depicted in Tables 1 and 2. The depth of the fairlead from the seabed is the same for both cases and is considered to be 457.2 m (1500ft). The vessel is considered to be subjected to several restoring forces which range from 88964.4N (20000lbs) to 889644N (200000lbs) at an interval of 88964.4N (20000lbs) (i.e., ten different restoring forces).

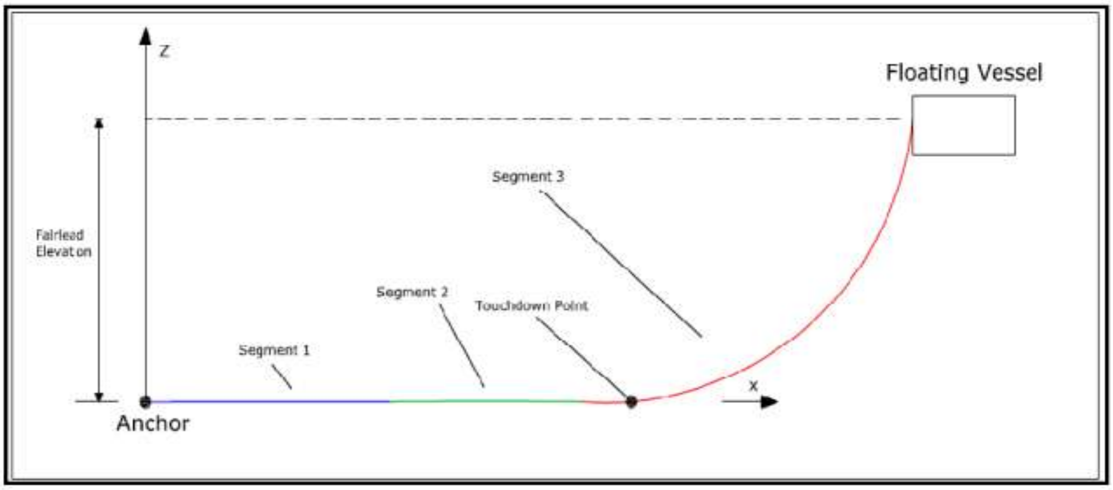

Figure 7: Diagrammatic Presentation of a Three Segment Mooring Line

Table 1:Properties of the Mooring Lines of Case 1(Source: [7])

Segment	**Length (m)**	**EA (N)**	**Weight per unit Length(N/m)**
1	304.8	93.4E10	11675.1
2	304.8	93.4E10	11675.1
3	304.8	93.4E10	11675.1

Table 2: Properties of the Mooring Lines of Case 2 (Source: Udoh, 2008)

Segment	Length (m)	EA (N)	Weight per unit Length(N/m)
1	304.8	93.4E10	11675.1
2	167.64	93.4E10	11675.1
3	198.12	93.4E10	11675.1

3.2 Mooring Lines Analysis Using the Elastic Catenary Model

The test cases considered in this work were tested by [7]. He tested the mooring lines using STAMOOSYS and LINANL. The same mooring lines are tested using the catenary elastic model developed in this work. The results obtained for the test case 1 is shown in Tables 3 to 6.

Table 3: Departure of the Mooring Line of Case 1

Restoring Force [N]	Departure [m] STAMOORSYS	LINANL	MATLAB	%Difference STAMOORSYS	LINANL
88964.40	486.27	486.25	486.25	-0.0041	0.0000
177928.80	505.13	505.08	505.1	-0.0057	0.0042
266893.20	520.32	520.29	520.32	-0.0007	0.0051
355857.60	533.30	533.25	533.28	-0.0036	0.0058
444822.00	544.56	544.59	544.65	0.0122	0.0103
533786.40	554.81	554.68	554.79	-0.0032	0.0202
622750.80	563.97	563.79	563.97	-0.0008	0.0312
711715.20	572.37	572.08	572.34	-0.0051	0.0456
800679.60	580.06	579.67	580.05	-0.0025	0.0648
889644.00	587.50	586.65	587.18	-0.0532	0.0898

Table 4: Fairlead Tension of the Mooring Line of Case 1

Restoring Force [N]	Fairlead Tension [N] STAMOORSYS	LINANL	MATLAB	%Difference STAMOORSYS	LINANL
88964.40	5426485.89	5426828.40	5426803.87	0.0059	-0.0005
177928.80	5515460.83	5515792.80	5515767.77	0.0056	-0.0005
266893.20	5604710.00	5604757.20	5604731.67	0.0004	-0.0005
355857.60	5693466.09	5694166.42	5693695.56	0.0040	-0.0083
444822.00	5782383.79	5783575.64	5782659.46	0.0048	-0.0158
533786.40	5871410.60	5872984.87	5871623.37	0.0036	-0.0232
622750.80	5960497.59	5962838.91	5960587.27	0.0015	-0.0378
711715.20	6049245.94	6053137.78	6049551.17	0.0050	-0.0593
800679.60	6138320.83	6143436.64	6138515.06	0.0032	-0.0802
889644.00	6227329.05	6234625.15	6227478.96	0.0024	-0.1148

Table 5: Suspended Length of the Mooring Line of Case 1

Restoring Force [N]	Suspended Length [m] STAMOORSYS	LINANL	MATLAB	%Difference STAMOORSYS	LINANL
88964.40	464.73	464.756	464.76	0.0056	-0.0008
177928.80	472.17	472.1962	472.19	0.0048	-0.0015
266893.20	479.51	479.5296	479.51	0.0008	-0.0033
355857.60	486.70	486.7626	486.73	0.0052	-0.0072
444822.00	493.81	493.904	493.83	0.0042	-0.0140
533786.40	500.82	500.9632	500.84	0.0032	-0.0248
622750.80	507.74	507.9462	507.74	0.0009	-0.0405
711715.20	514.53	514.8651	514.56	0.0058	-0.0602

800679.60	521.27	521.7231	521.29	0.0032	-0.0832
889644.00	527.91	528.5323	527.93	0.0033	-0.1141

Table 6: Length on the Sea Floor of the Mooring Line of Case 1

Restoring Force [N]	Length on Sea Floor [m]			%Difference	
	STAMOORSYS	LINANL	MATLAB	STAMOORSYS	LINANL
88964.40	449.67	449.58	449.64	-0.0058	0.0142
177928.80	442.14	442.20	442.21	0.0152	0.0016
266893.20	434.89	434.86	434.89	-0.0009	0.0058
355857.60	427.70	427.64	427.67	-0.0059	0.0078
444822.00	420.59	420.47	420.57	-0.0074	0.0220
533786.40	413.58	413.44	413.56	-0.0039	0.0276
622750.80	406.66	406.45	406.66	-0.0011	0.0457
711715.20	399.87	399.53	399.84	-0.0075	0.0689
800679.60	393.13	392.68	393.11	-0.0042	0.0964
889644.00	386.49	383.13	386.47	-0.0045	0.7434

The results are compared by simple percentage difference to obtain the deviation of the results when compared to STAMOORSYS and LINANL. The three methods agree with a high degree acceptance. The maximum percentage deviation of the departure computed when compared to STAMOORSYS is 0.0122% and 0.1148% when compared with LINANL. A significant deviation is observed in the computation of the length on sea floor by LINANL at the maximum restoring force. Udoh (2008) explained that this deviation (0.734%) may be as a result of convergence of the iterative procedure of LINANL at significant error. The results are also plotted against the restoring forces. The plot of departure, fairlead tension, suspended length and the length on sea floor of the mooring line of test case 1 against the restoring forces are depicted in Figures 8-11 respectively.

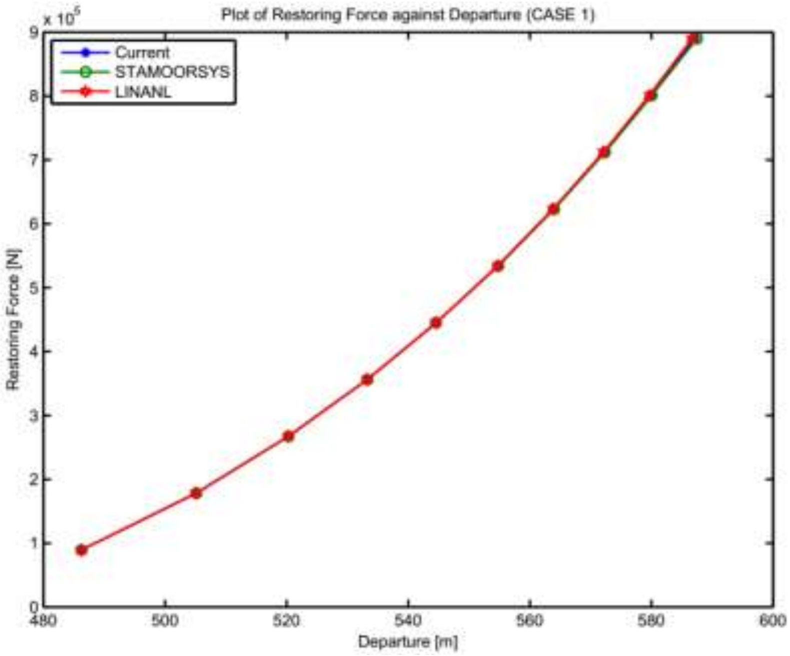

Figure 8: Departure of Case 1

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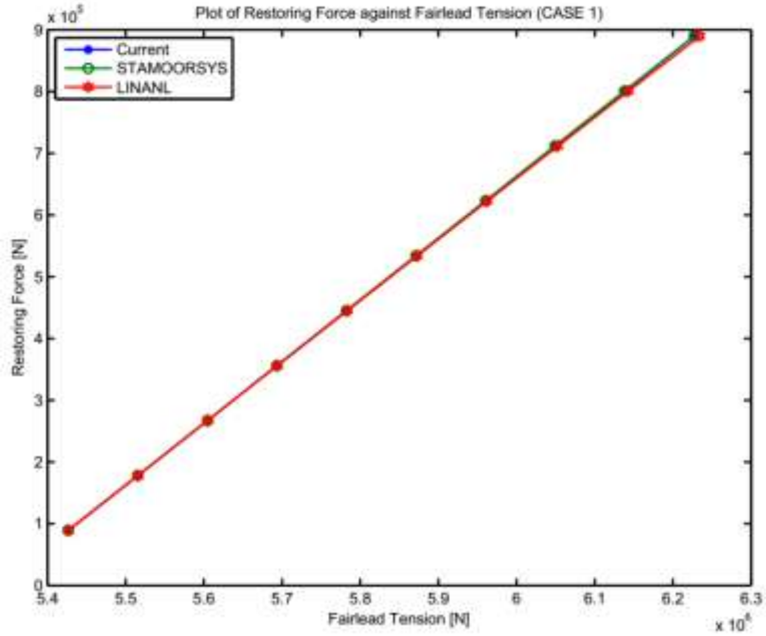

Figure 9: Fairlead Tension of Case 1

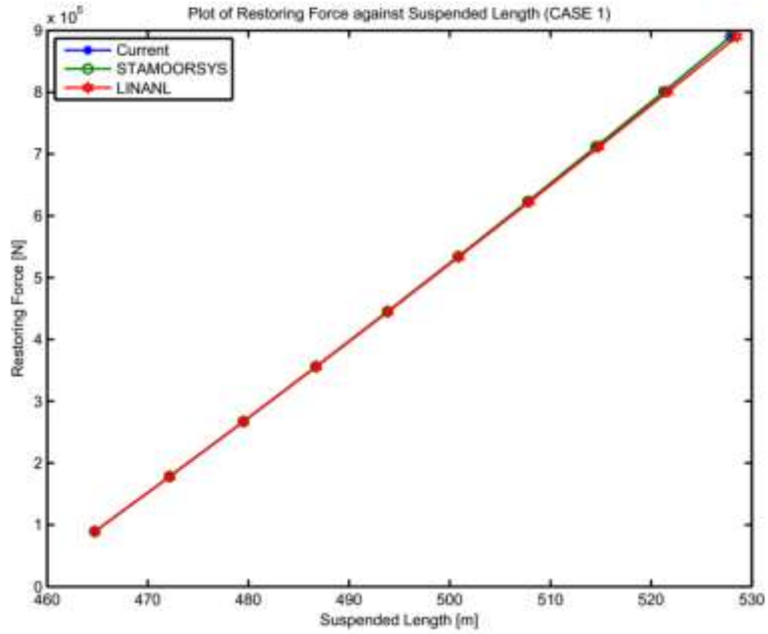

Figure 10: Suspended Length of Case 1

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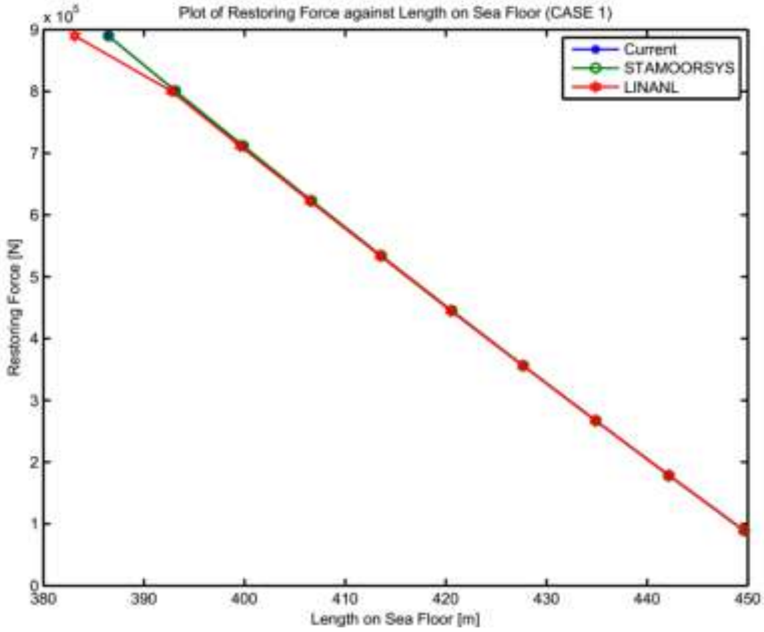

Figure 11: Length on Sea Floor of Case 1

Similarly, the second case (case 2) was analyzed and presented. As shown in Table 2, the length of the mooring line in test case 1 is reduced. The length of segment 2 is reduced from 304.8m to 167.64m and that of segment 2 is reduced to 198.12m. The results obtained from the comparisons of the various program is presented in Tables 7 – 10 and Figures 12 - 15. Computed departures, tensions and line configurations atequilibrium agree very strongly. The suspended lengths and lengths on sea floorcomputed by the programs show the results to be physically correct.

Table 7: Departure of the Mooring Line of Case 2

Restoring Force [N]	Departure [m]			%Difference	
	STAMOORSYS	LINANL	MATLAB	STAMOORSYS	LINANL
88964.40	242.44	242.48	242.41	-0.0124	-0.0289
177928.80	261.29	261.28	261.26	-0.0111	-0.0073
266893.20	276.48	276.52	276.48	-0.0013	-0.0158
355857.60	289.28	289.52	289.44	0.0556	-0.0273
444822.00	300.83	300.93	300.81	-0.0079	-0.0411
533786.40	310.97	311.14	310.95	-0.0058	-0.0604
622750.80	320.13	320.34	320.13	-0.0013	-0.0669
711715.20	328.53	328.79	328.50	-0.0088	-0.0880
800679.60	336.22	336.56	336.21	-0.0043	-0.1054
889644.00	343.35	343.78	343.34	-0.0037	-0.1289

Table 8: Fairlead Tension of the Mooring Line of Case 2

Restoring Force [N]	Fairlead Tension [N]			%Difference	
	STAMOORSYS	LINANL	MATLAB	STAMOORSYS	LINANL
88964.40	5426490.16	5426828.4	5426803.87	0.0058	-0.0005
177928.80	5515460.83	5515347.98	5515767.77	0.0056	0.0076
266893.20	5604710.00	5604312.38	5604731.67	0.0004	0.0075

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355857.60	5693466.09	5692831.96	5693695.56	0.0040	0.0152
444822.00	5782383.79	5780906.71	5782659.46	0.0048	0.0303
533786.40	5871410.60	5869426.29	5871623.37	0.0036	0.0374
622750.80	5960497.59	5957501.05	5960587.27	0.0015	0.0518
711715.20	6049245.94	6045575.8	6049551.17	0.0050	0.0657
800679.60	6138320.83	6133650.56	6138515.06	0.0032	0.0792
889644.00	6227329.05	6221725.31	6227478.96	0.0024	0.0924

Table 9:Suspended Lengthof the Mooring Line of Case 2

Restoring Force [N]	Suspended Length [m]			%Difference	
	STAMOORSYS	LINANL	MATLAB	STAMOORSYS	LINANL
88964.40	464.73	464.75	464.76	0.0056	0.0013
177928.80	472.17	472.17	472.19	0.0048	0.0048
266893.20	479.51	479.47	479.51	0.0008	0.0092
355857.60	486.7	486.64	486.73	0.0052	0.0175
444822.00	493.81	493.7	493.83	0.0042	0.0265
533786.40	500.82	500.64	500.84	0.0032	0.0391
622750.80	507.74	507.48	507.74	0.0009	0.0521
711715.20	514.53	514.23	514.56	0.0058	0.0641
800679.60	521.27	520.87	521.29	0.0032	0.0799
889644.00	527.91	527.42	527.93	0.0033	0.0961

Table 10: Length on Sea Floor of the Mooring Line of Case 2

Restoring Force [N]	Length on Sea Floor [m]			%Difference	
	STAMOORSYS	LINANL	MATLAB	STAMOORSYS	LINANL
88964.40	205.83	205.8	205.80	-0.0127	0.0019
177928.80	198.39	198.39	198.37	-0.0115	-0.0110
266893.20	191.05	191.09	191.05	-0.0021	-0.0262
355857.60	183.86	183.92	183.83	-0.0136	-0.0413
444822.00	176.75	176.86	176.73	-0.0118	-0.0636
533786.40	169.74	169.92	169.72	-0.0094	-0.0952
622750.80	162.82	163.08	162.82	-0.0027	-0.1284
711715.20	156.03	156.34	156.00	-0.0192	-0.1652
800679.60	149.32	149.69	149.27	-0.0312	-0.2024
889644.00	142.65	143.14	142.63	-0.0122	-0.2465

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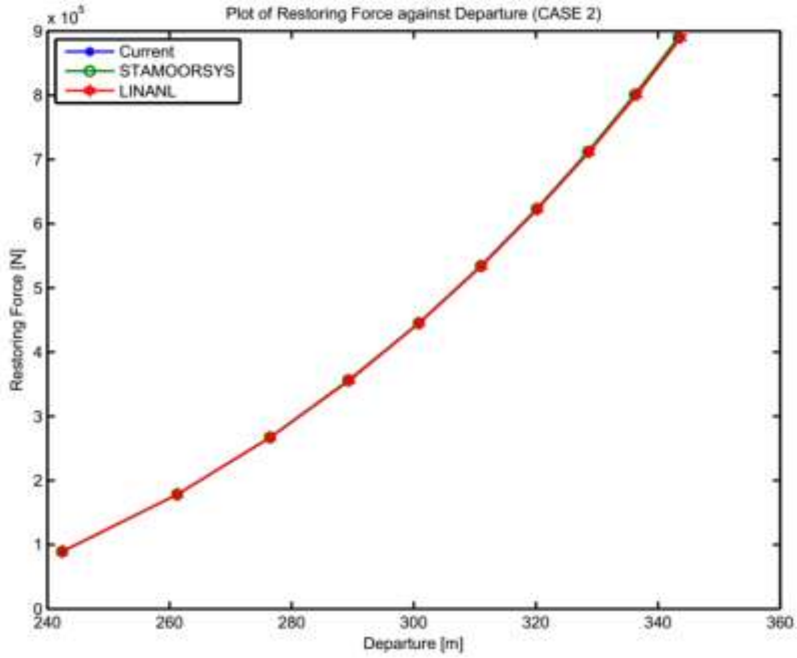

Figure 12: Departure of Case 2

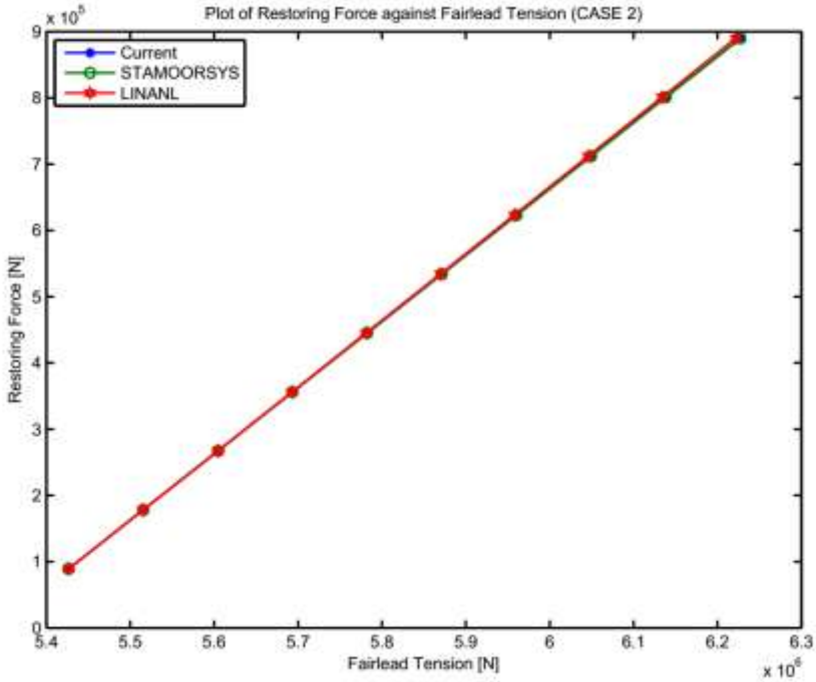

Figure 13: Fairlead Tension of Case 2

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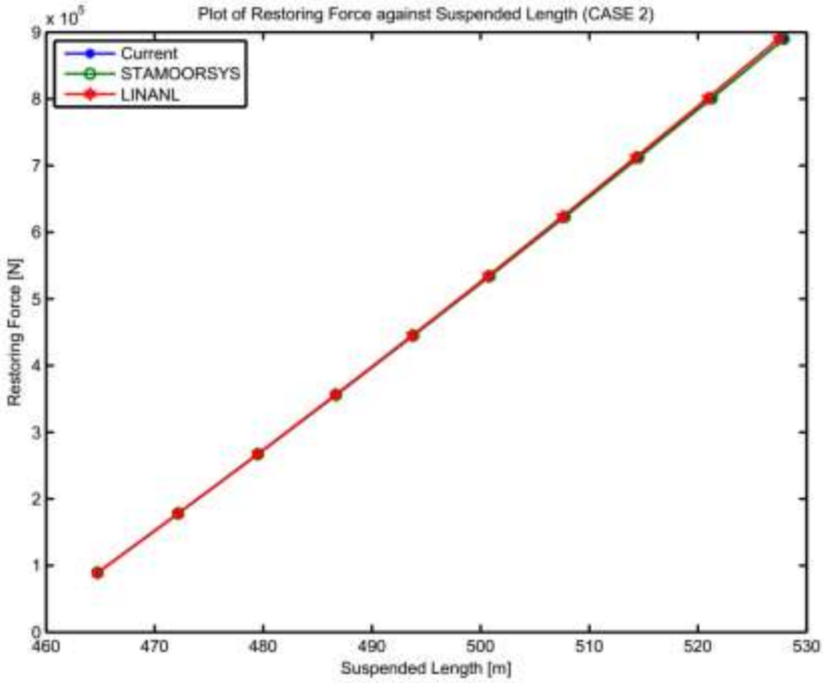

Figure 14: Suspended Length of Case 2

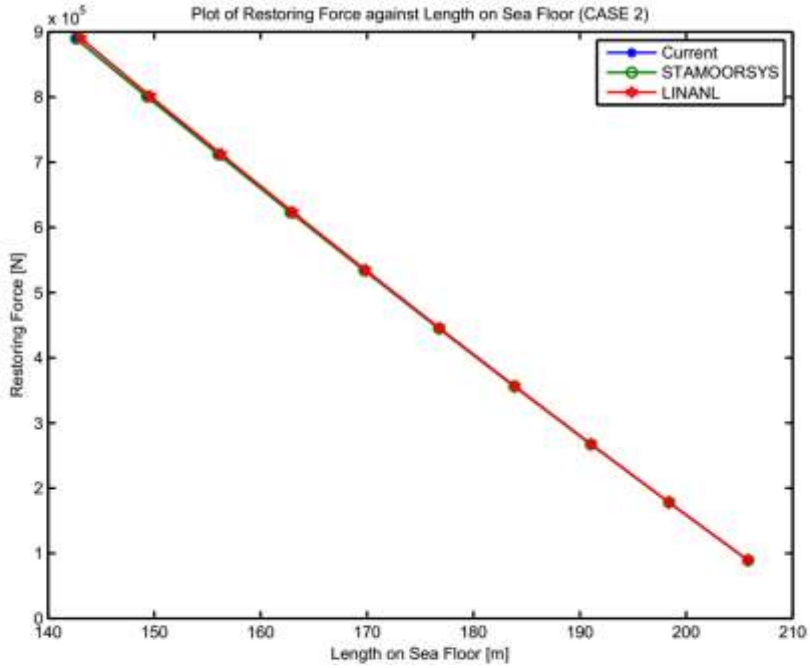

Figure 15: Length on Sea Floor of Case 2

3.3 Comparison of the Elastic and Inelastic Catenary Models

Further analysis was carried out on the mooring line of test case 2. The line was analyzed using the elastic and inelastic catenary mooring models. The results obtained from the analysis are presented in Table 11 and Figures 10 through 13. From the graphs and the table, it is very clear that both the elastic and inelastic model gives a good prediction of the static behaviour of the mooring line considered in this analysis. The departure, suspended length and length on the sea floor are predicted to be the same in both the elastic and inelastic model. This is as true because, the load causes insignificant stretching of the line.

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Table 11: Comparison of the Elastic and Inelastic Catenary Model

Restoring Force [N]	Departure [m] Elastic	Departure [m] Inelastic	Fairlead Tension [N] Elastic	Fairlead Tension [N] Inelastic	Suspended Length[m] Elastic	Suspended Length[m] Inelastic	Length on Sea Floor[m] Elastic	Length on Sea Floor[m] Inelastic
88964.40	486.25	486.25	5426803.87	5426820.12	464.76	464.76	449.64	449.64
177928.80	505.10	505.10	5515767.77	5515784.52	472.19	472.19	442.21	442.21
266893.20	520.32	520.31	5604731.67	5604748.92	479.51	479.52	434.89	434.88
355857.60	533.28	533.28	5693695.56	5693713.32	486.73	486.73	427.67	427.67
444822.00	544.65	544.64	5782659.46	5782677.72	493.83	493.83	420.57	420.57
533786.40	554.79	554.79	5871623.37	5871642.12	500.84	500.84	413.56	413.56
622750.80	563.97	563.96	5960587.27	5960606.52	507.74	507.75	406.66	406.65
711715.20	572.34	572.34	6049551.17	6049570.92	514.56	514.56	399.84	399.84
800679.60	580.05	580.04	6138515.06	6138535.32	521.29	521.29	393.11	393.11
889644.00	587.18	587.18	6227478.96	6227499.72	527.93	527.93	386.47	386.47

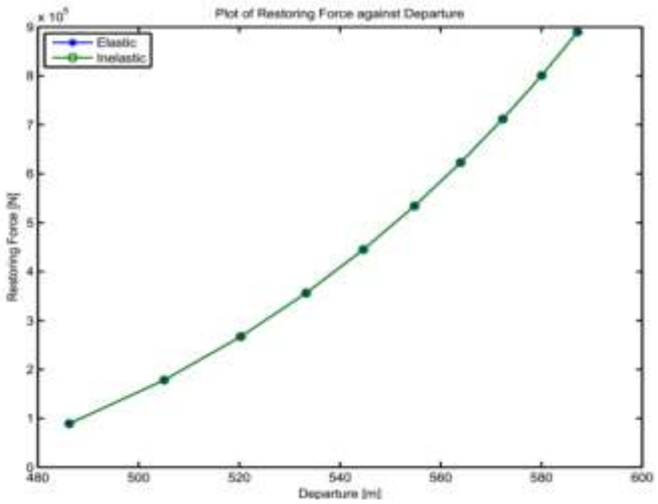

Figure 16: Departure of Elastic and Inelastic Catenary Mode

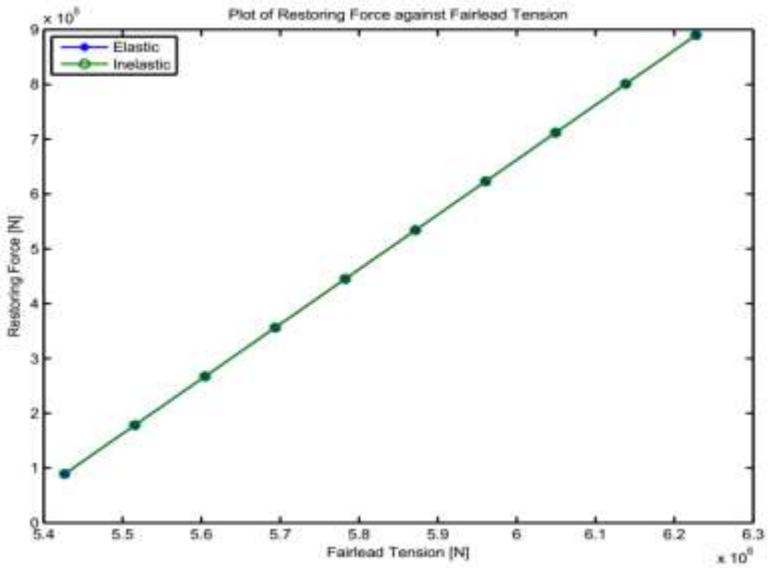

Figure 17: Fairlead Tension of Elastic and Inelastic Catenary Mode

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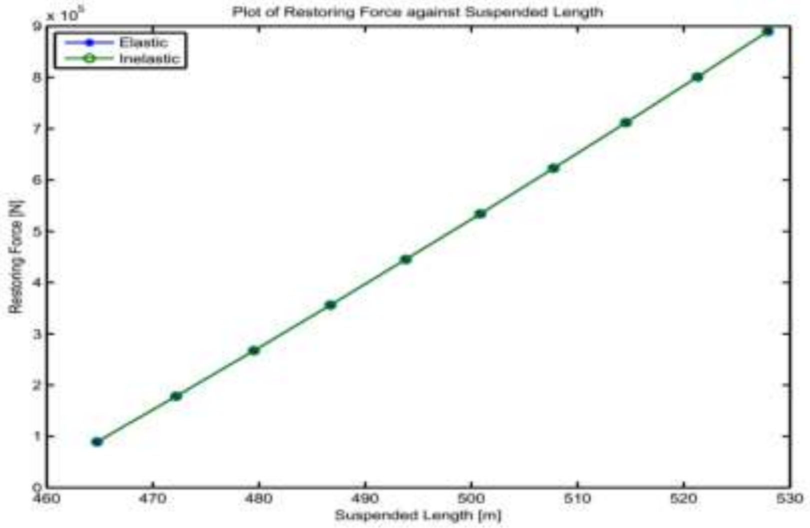

Figure 18: Suspended Length of Elastic and Inelastic Catenary Mode

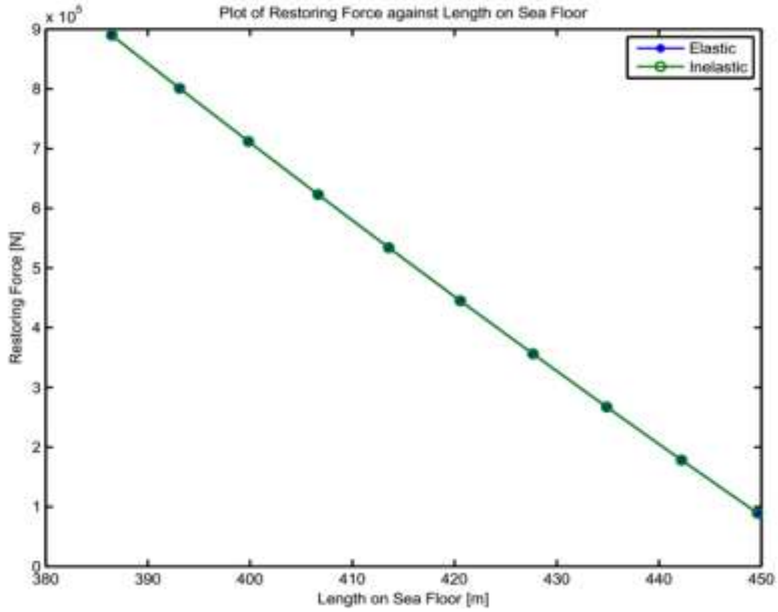

Figure 19: Length on Sea Floor of Elastic and Inelastic Catenary Model

3.4 Mooring Stiffness of the Mooring Line

Further analysis were carried out on the mooring line considered in this work to study the variation of the mooring line stiffness. The variation of the line stiffness with respect to the ratio of horizontal tension to immersed weight per unit length (a) was plotted as shown in Figure 20. The study is carried out for a constant water depth of 457.2 m as defined earlier for the mooring lines studied in this work. As can be seen from Figure 20, the mooring stiffness increases with increase in the ratio of horizontal tension to immersed weight per unit length.

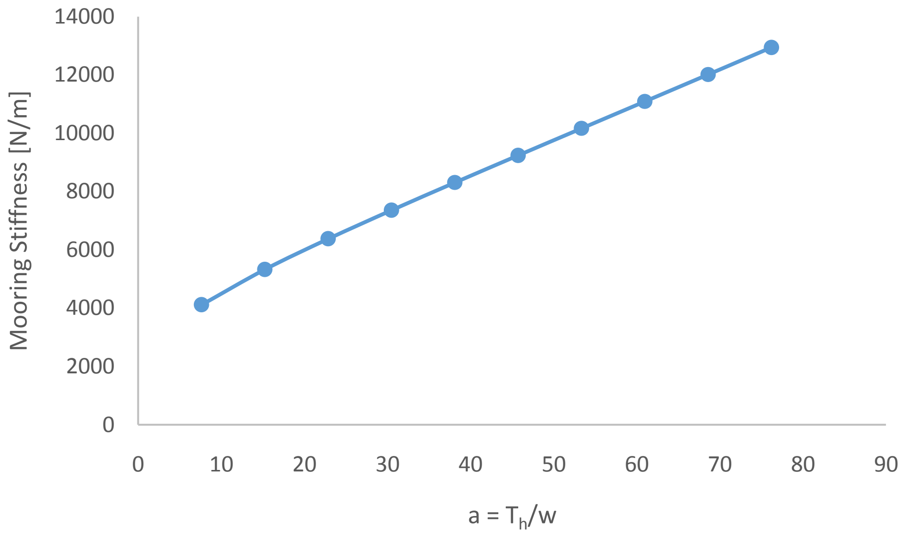

Figure 20: Variation of Mooring Stiffness with Respect to $\frac{T_h}{w}$

Similarly, the mooring line stiffness was also studied to understand its variation with respect to water depth. In this instance, the value of ratio of horizontal tension to immersed weight per unit length is kept constant. A case of constant value of the ratio of horizontal tension to immersed weight per unit length were considered. The result obtained from the analysis is shown in Figure 21. The stiffness coefficient decreases with increase in water depth. As the water depth increases, the value of the stiffness coefficient tend to be constant.

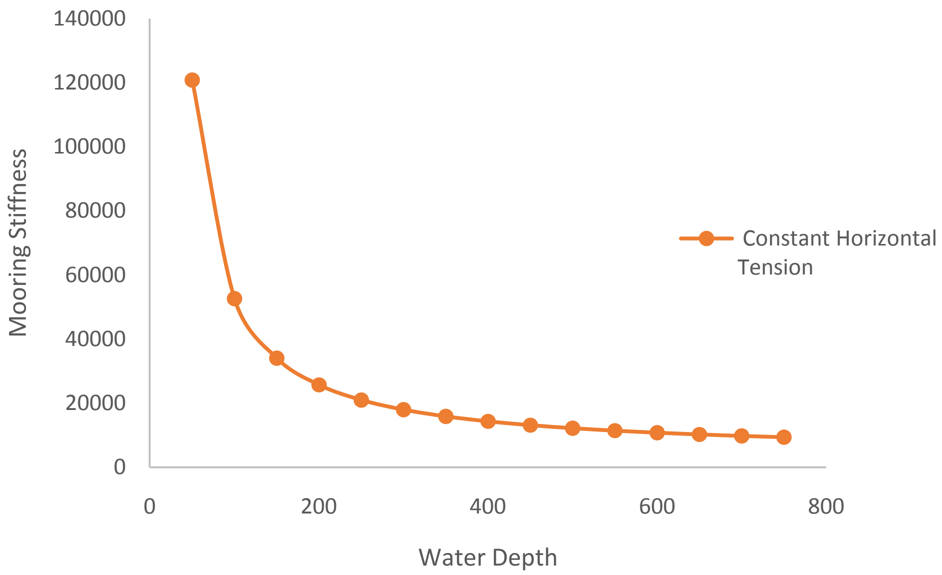

Figure 21: Variation of Mooring Stiffness with Respect to Water Depth

From the graph (21) above, the water suspending the mooring line is as a result of the buoyancy, reason being that at high depth the stiffness is reduced.

IV. CONCLUSION AND RECOMMENDATIONS

4.1 Conclusion

In this dissertation, the static analysis of mooring systems for stationkeeping of offshore structures is considered. The type of mooring system considered in this work is the single component, single line mooring. The catenary equations were applied for the purpose of the static analysis of the mooring lines.Both the elastic and inelastic cables were considered in this work. The catenary equations were used to model the case of elastic cables as well as the inelastic cables. The major response of the mooring line that is of importance was analyzed. In the static analysis of mooring lines, the main parameters of importance are the departure (the horizontal distance from the anchor to the fairlead), the fairlead tension, the suspended length of the line and the touchdown length. The catenary model developed in this work has the capability of analyzing the aforementioned parameters given the weight per unit length of the line, length of the line and the restoring force of the moored vessel in the case of inelastic cable line whereas, in the case of elastic length, an additional parameter (the line rigidity) is required.

The developed catenary model for both elastic and inelastic mooring lines is implemented in MATLAB. The developed MATLAB program was used to analyze two test cases of mooring lines. The two test case were also analyzed using two different software (STAMOORSYS and LINANL) to validate the accuracy of the developed MATLAB program in this dissertation.

The two software also have the capabilities of analyzing the various parameters of importance in mooring line design as stated above. The results obtained from each of the programs are tabulated and plotted against the restoring forces of the vessel. Comparison is also made between the results obtained from the MATLAB program developed in this work and that of STAMOORSYS and LINANL. The percentage deviation of the results is very minimal for the various parameters computed. This to say, the MATLAB program developed in this work showed a good result for predicting the static response of mooring lines when compared with STAMOORSYS and LINANL.

Further analysis was carried out to compare the elastic and inelastic model. It is observed that when the inelastic model was used to analyze the test case 1 of the mooring line in this work, the results obtained agrees with that of the elastic model. This observation was attributed to the fact that the mooring lines considered in this work have high values of modulus of rigidity and can be considered to be rigid. Therefore, the catenary model for inelastic cable can be used to analyze such cables.

4.2 Contributions to Knowledge

This work presents a detailed procedure of statically analyzing single line single component mooring system. The MATLAB program developed in this work can be used to aid the design and analysis of mooring lines (both elastic and inelastic cables). This work also present methods of analyzing the mooring stiffness with respect to the water depth as well as the ratio of horizontal tension to immersed weight per unit length.

4.3 Recommendations

The following recommendations are deemed necessary from this research work:

1. Static analysis of mooring systems ignores the effect of hydrodynamic force caused by wave and current. Therefore, it is recommended that, in the design of statically mooring lines, factors of safety should be considered to account for the hydrodynamic force caused by wave and current.
2. The effect of dynamic forces is also ignored in the development of the models because we assumed the line to be static. This is not the case when the vessel is subjected to motion resulting from winds and wave current. Therefore, it is recommended that, dynamic models be developed and included in the MATLAB program developed in this work to properly analyze and simulate the response of mooring systems.

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